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RESEARCH ARTICLE

Local taper functions for natural black pine forests in the Northwestern Rhodopes in Bulgaria Local taper functions for black pine

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Abstract

Unlike the heavily deforested Eastern or Low Rhodopes, the Western or High Rhodopes are one of Bulgaria's main forestry areas, where black pine is widespread and economically important. Based on stem analyzes, an average taper curve for black pine stems in the Northwestern Rhodopes was determined. The method of relative taper curves (*echte Ausbauchungsreihen*) was used. Taper models based on spline functions and power functions were fitted. A comparison with the official assortment table of Nedyalkov was made. The systematic deviation of the sample stems from the long-used assortment table justifies the development of a local black pine table.

Keywords

Assortment tables, merchantable volume, assortment standards

Introduction

The Northwestern Rhodopes and more precisely the forestry offices Chepino and Alabak (Velingrad) are one of the significant habitats of natural black pine forests in Bulgaria. Black pine (*Pinus nigra* J.F.Arnold) is widespread in Bulgaria, but mainly in plantations - 90% of black pine stands are artificial. Its natural stands have survived only in the mountains of Southwest Bulgaria, including the Rho-

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dopes. Outside of the Southwest, a few relict stands of natural black pine have survived only in the Western Stara Planina. However, in Southwest Bulgaria, which is a mountainous land cut by the Struma and Mesta valleys, natural black pine is an important economic species, with 35,000 ha of forested area and 8,000,000 cubic meters of stock. A large part of the natural black pine forests are mature and over-mature (42%), which makes the question of the exact determination of their stock relevant.

The official assortment tables for natural black pine were created long ago (Nedyalkov, 1954). Initially, they were also used for artificial plantations. At the very beginning of the 1980s, tables were created for artificial black pine plantations (Tsakov, 1981) and Nedyalkov's tables remain to be used only for natural stands (Krastanov, Raykov, 2004). Using the same volume and growth tables for natural forests and artificial plantations is common in Germany (Schober, 1952, 1987). This suggests that stem shape variation should be investigated with modern means at least in the case of black pine (Petrin, 2022).

The aim of the present work is to study the stem shape of the black pine in the Northwestern Rhodopes in preparation for the development of a local assortment table.

Materials and methods

Methods

In the present work we use taper curves of the type $q = q(h)$, where q is the cross-sectional area of the stem without bark at a height h above the ground. The height of the tree will be denoted by H (Fig. 1). The volume of the stem is simply the integral of the curve, and the mean cross-section is the ratio $\overline{q} = v/H$.

In Bulgaria it is accepted to present the value q_h as a percentage of mean cross-sectional area. This leads to curves that set the value $\varphi = q_h / \overline{q}$ as a function of relative height $x = h / H$. This presentation in Bulgaria is known from prof. Sirakoff's publications (Sirakoff, 1958, Mihov, 2005). It is equivalent to the *echte Ausbauchungsreihen* of Hohenadl (Kramer, 1987), widely used by Zakharov's students in Russia and also by some authors in Bulgaria (Dimitrov, 2003), but is associated with simpler formulas. The exact definition is (Mihov, Markov, 1998).

$$\varphi(x) = \frac{q(xH)}{\overline{q}} \quad (1)$$

wherein

φ - taper function,

x - the relative height above ground, $0 \le x \le 1$.

The representation of the stem curve in relative units greatly reduces the natural variation of curves and allows all trees of one tree species not to be

represented with one averaged such curve. Based on this curve, the volume and assortment of each stem can be determined as follows:

stem volume without bark v

$$v = \frac{\pi d^2}{4} \frac{H}{\varphi(1.3/H)} \quad (2)$$

stem diameter at h m above ground

$$d(h) = d\sqrt{\frac{\varphi(h/H)}{\varphi(1.3/H)}} \quad (3)$$

height above ground of top diameter t

$$h = H\,\phi^{-1}\left[\frac{t^2}{d^2}\varphi(1.3/H)\right] \quad (4)$$

stem volume v_h between h and the ground

$$v_h = v\int_0^{h/H}\varphi \quad (5)$$

stem taper at h m above ground

$$a_h = d(h) - d(h+1) \quad (6)$$

Linear combinations of power functions were used to model the taper functions

$$y = ax^n + bx^m + cx^k \quad (7)$$

wherein

a, b, c, m, n, k - regression coefficients

$x = 1 - h/H$,

and also spline functions of the type

$$y = c\left(a\,x^n + a_1[x-n_0]^{1.5} + a_2[x-n_0]^3 + a_3[x-n_1]^3 + a_4[x-n_2]^3\right), \quad (8)$$

wherein

$c, n, n_0, n_1, n_2, a, a_1, a_2, a_3, a_4$ - regression coefficients,

$x = 1 - h/H$

and the angular brackets mean setting to 0 the negative numbers,

$$[z] = \begin{cases} z, & \text{for } z \geq 0 \\ 0, & \text{for } z < 0 \end{cases} \quad (9)$$

Materials

To study the stem shape of black pine, 23 stems were used, aged from 52 to 138 years, with diameters from 23 to 62 cm and height from 20 to 33 m, harvested in the area of the state forestry offices Chepino and Alabak (the latter is also known as Velingrad). Their shape was determined by measuring two-meter sections in the usual way.

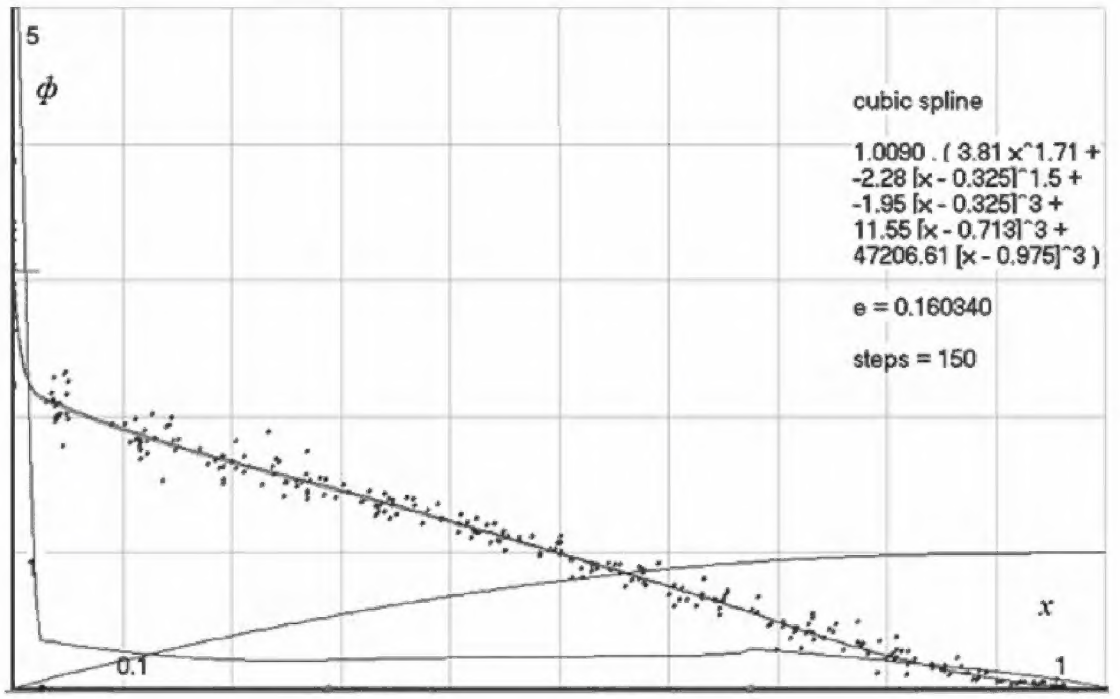

Figure 1. Taper function obtained from NW Rhodopes stem analyses, spline function.

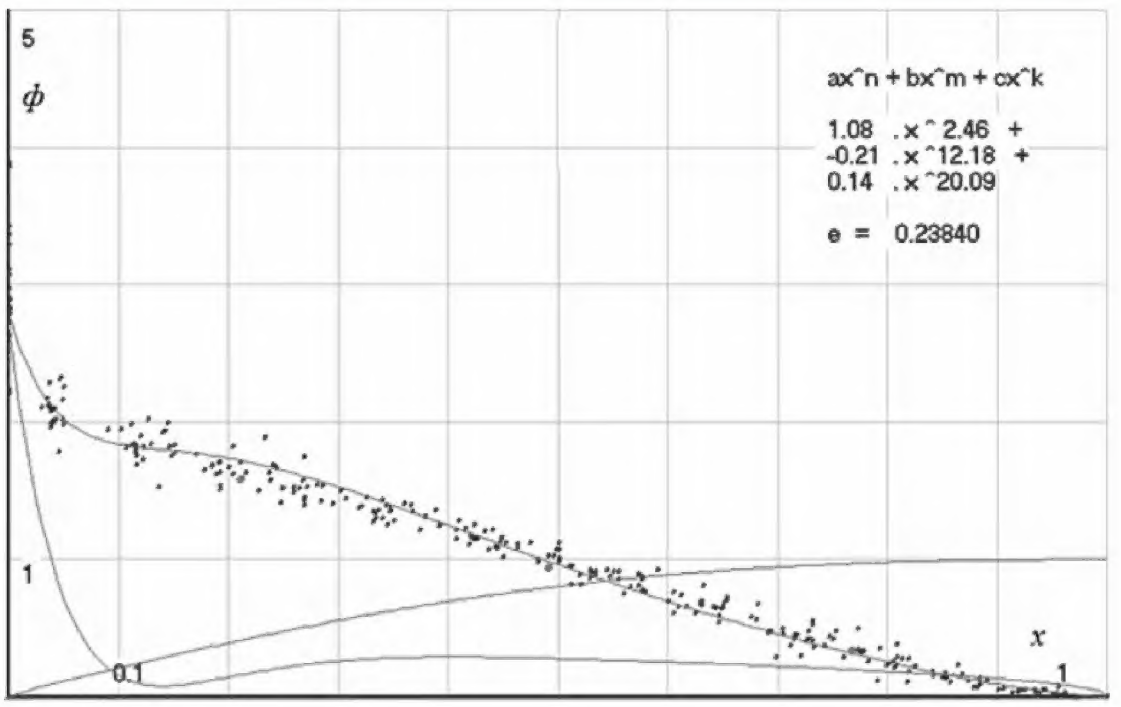

Figure 2. Black pine taper function obtained from Northwestern Rhodopes stem analyses, linear combination of power functions.

Results

The results of modeling the stem shape obtained from the stem analyzes are presented in Figures 1 and 2. The regression coefficients are given in Table 1. The quality of the approximation was estimated by mean quadratic error e (the standard deviation of the curve from the experimental points). The accuracy of the approximation is better with the spline function, which also visually passes better through the data.

To make a comparison with the official assortment table of Nedyalkov, the latter was presented with the same spline function. The modeling accura-

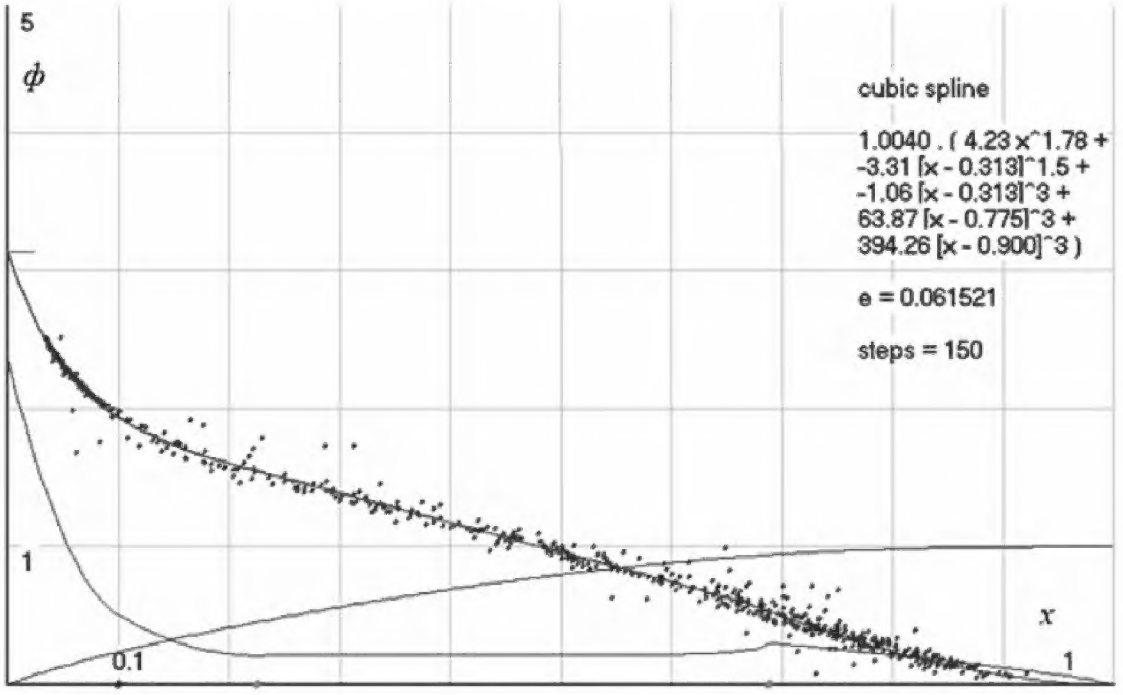

Figure 3. Black pine taper function obtained back form assortment table of Nedyalkov, spline function.

Table 1. Regression coefficients

Spline Functions			Power Functions	
Coefficients	**Nw Rhodopes Data**	**Official Assortment Table**	**Coefficients**	**Nw Rhodopes Data**
c	1.009042	1.001763	a	1.076217
a	3.812676	2.94522	n	2.459603
n	1.705873	1.5	b	-0.21208
a_1	-2.28	-1.65857	m	12.18249
n_0	0.325	0.3625	c	0.13586
a_2	-1.94511	-0.36937	k	20.08554
a_3	11.55301	63.98413		
n_1	0.7125	0.775		
a_4	47206.61	387.3285		
n_2	0.975	0.9		

cy was even higher. The resulting curve is the back-calculated taper function from which the table has been calculated since it is certain that Nedyalkov has applied the same method.

A comparison of the two models is made in Figure 4. It shows a significant discrepancy between the two models in the ground part of the stem, which is especially important for determining the stock. From formula (2) it is obvious that the official table systematically underestimates the volume of the stems at least in our sample.

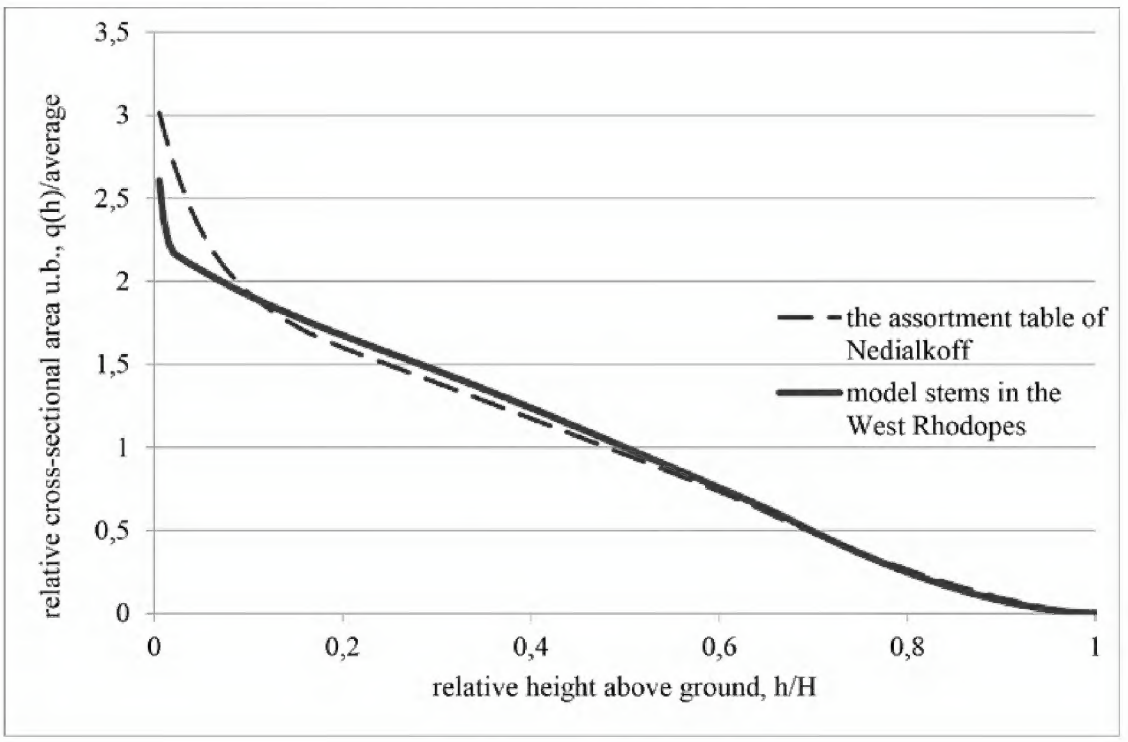

Figure 4. Comparison of Nedyalkov's official table for black pine with our measurements in the NW Rhodopes.

Discussion

In North America, assortment tables are often fitted directly from measured data without fitting a taper curve first. However, the use of taper curve models allows for a flexible response to market demand and a smooth transition to the European standard for defining roundwood assortments.

The method used in Bulgaria for taper modeling (Sirakov, 1947a,b; Sirakoff, 1958; Prodan, 1961) is equivalent to the *echte Ausbauchungsreihen*, used by many authors, e.g. Laasasenaho (1982). Without citing it, many authors use equivalent methods (Benett, Swindell, 1972; Curtis et al, 1968). However, the method of the Swiss National Forest Inventory (Brassel et al., 2001) and the *unechte Ausbauchungsreihen* used by Schober (1952) are irreducible to it. Sirakoff himself (Sirakov, 1947a,b) considered the need to treat the variations of the stem shape with a linear combination of curves of the type (1).

In the present work, we find a systematic deviation of the test stems from the long-used assortment table, which is sufficient to justify the need to develop a local table of black pine and to continue research on the stem form of this species.

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